Introduction to Measurement Error: Prevalence, Impact and Adjustments

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  - Discrepancies between the ‘true’ and the observed value
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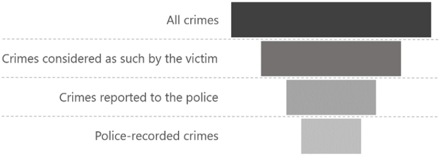
  - The consequence of a poorly defined construct and/or an imperfect measurement process
- Examples in the Social Sciences
  - Elusive constructs, loosely defined:
    e.g. happiness, ethnicity, political decentralisation
  - Subjectively elicited data:
    e.g. survey data, affected by memory failures (*when was the last time you went to a pub?*), social desirability (*for how long have you been unemployed?*)
  - Administrative/official data used as proxies:
    e.g. using earnings to measure poverty, or measuring violent crime from police records

The Recounting Crime project

- Police recorded crime statistics are deeply flawed



  - Under-reporting/under-detection of crime
  - Recording inconsistencies across forces

Implications and Solutions

- Why does it matter?
  - We cannot describe reality accurately

    e.g. What is the true prevalence of crime in Hong Kong? Has it grown from last year? Is it higher than in Macau?
  - But also, our causal inferences will be biased

    e.g. Does unemployment affect crime? Does crime affect mental health?

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    e.g. Does unemployment affect crime? Does crime affect mental health?
- There are ways to anticipate the prevalence and impact of measurement error
  - And to some extent adjust for it
  - But to do so we first need to define these errors formally using measurement error models

Defining Measurement Error Formally

- The classical measurement error model (random errors)
  - $\overbrace{X^*}^{observed} = \overbrace{X}^{true\ value} + \overbrace{U}^{noise}$
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- Examples:
  - Recording inconsistencies due to police officers' recall errors
  - Results from a math test
- Only the variance is affected
  - $\sigma^2_{X^*} = \sigma^2_X + \sigma^2_U$; but the mean is unaffected since $E(U) = 0$

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- Examples:
  - Crime reported to the police
  - Self-reported xenophobia, sexual partners, etc.

Multiplicative Errors

- So far we have assumed that errors are independent
- What if the error is proportional to the true value of the quantity being measured?
  - E.g. memory failures in reporting counts;

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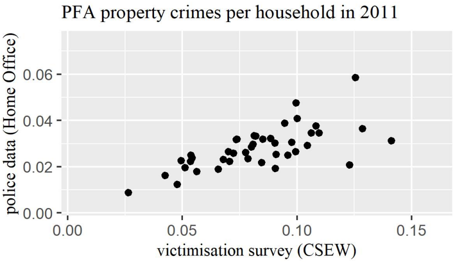
  - E.g. memory failures in reporting counts;
    How many alcoholic drinks did you have last week?
- These can be better specified using a multiplicative rather than an additive model
  - I.e., as $X^* = X \cdot U$, rather than $X^* = X + U$

Multiple Error Mechanisms

- Often variables are affected by multiple measurement error mechanisms
- This is how we define measurement error in police data
  - *systematic*, since not all crime is reported to the police
  - *random*, subject to variability across areas, as a result of the different recording practices across police forces
  - *multiplicative*, errors seem proportional to the true extent of crime in the area

Multiplicative Errors: Crime Rates



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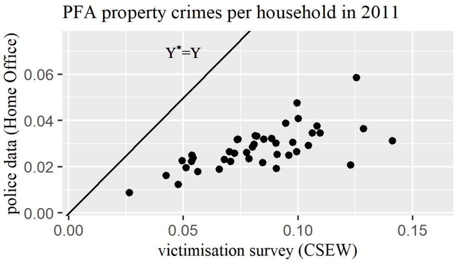

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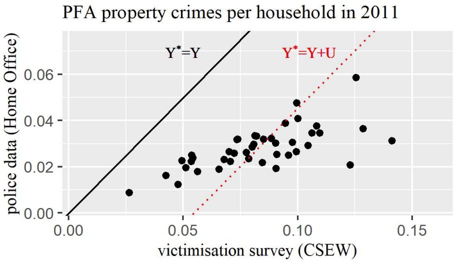

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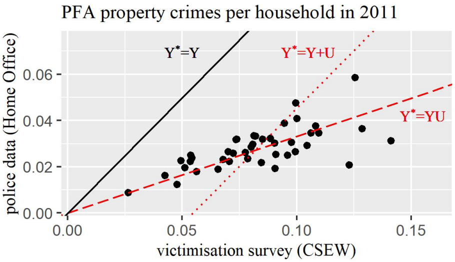

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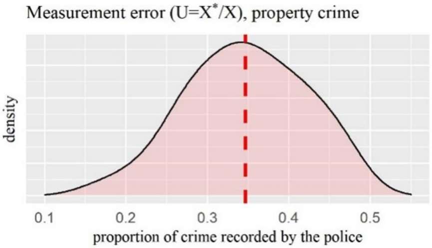

The Impact of Measurement Error

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  - Random errors affect measures of dispersion, systematic errors affect measures of centrality
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- We have seen how different forms of measurement error can affect univariate stats
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- But how does measurement error affect estimates from multivariate (regression) models?
- Assuming only one variable is prone to measurement error, its impact will depend on:
  1. the outcome model (whether linear, Poisson, etc.)
  2. the measurement error model (additive, random, etc.)
  3. where is the affected variable introduced in the model (as a response or an explanatory variable)

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- Question: Will β be biased in any of those scenarios?

Classical Error on the Response

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    $Y = \alpha + \beta X + (\epsilon - U)$
  - The measurement error is absorbed by the model's error term, affecting precision, but leaving regression coefficients unbiased
  - We can see this effect using simulated data

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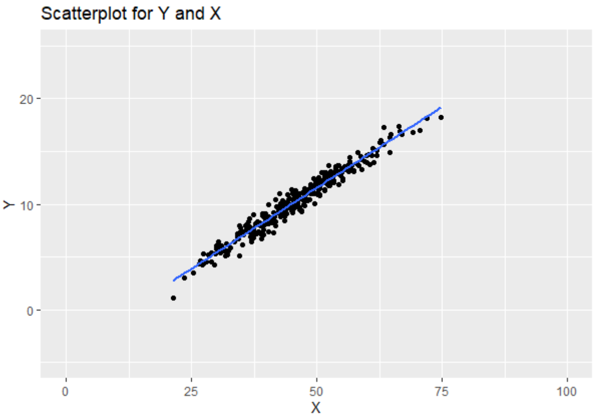

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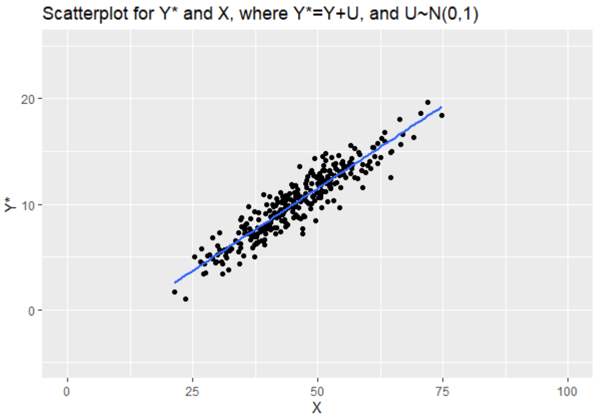

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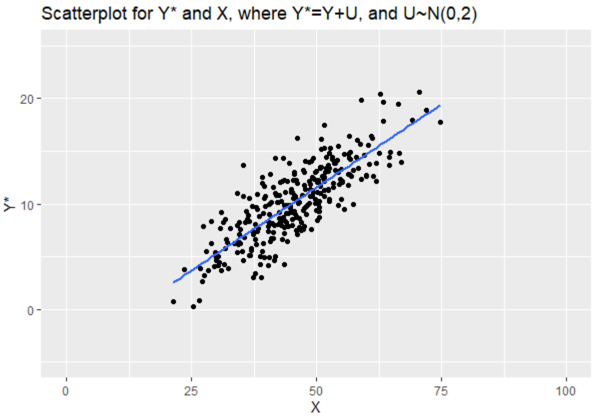

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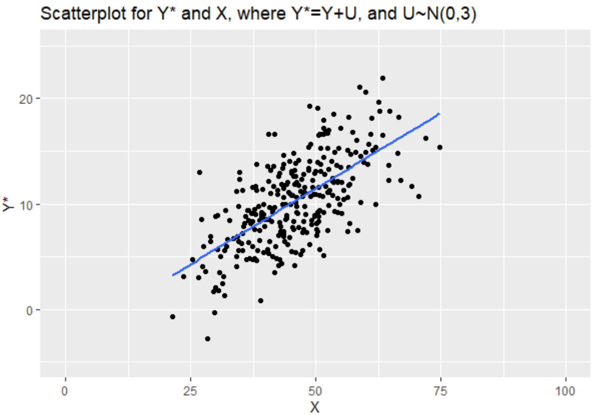

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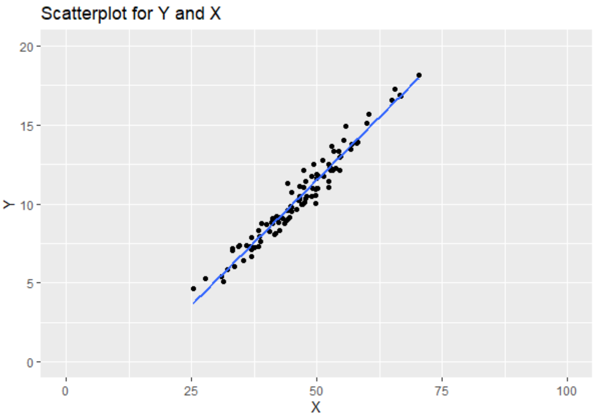

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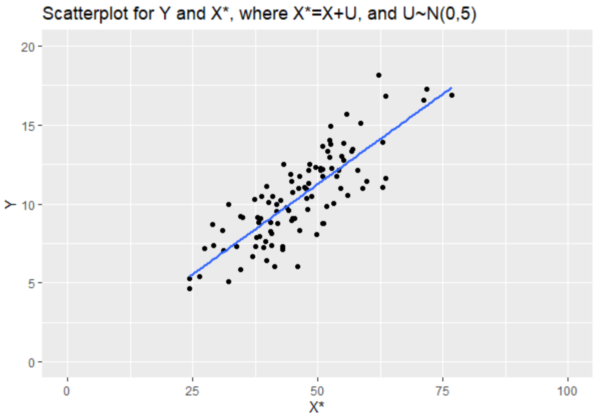

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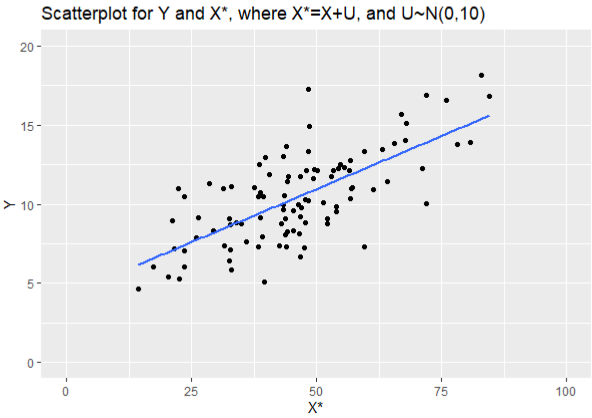

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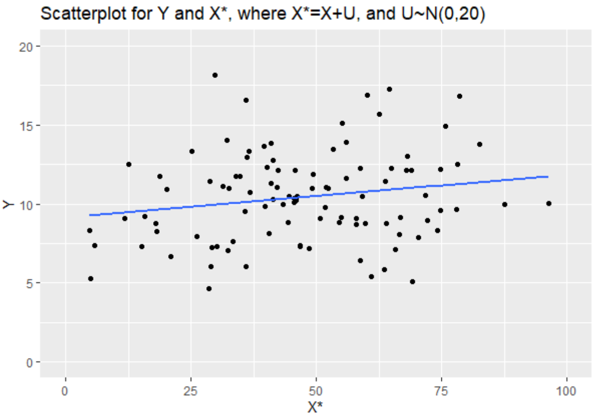

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    $Y \cdot U = \alpha + \beta X + \epsilon$

    $$Y = \frac{\alpha + \beta X + \epsilon}{U}$$
  - All regression coefficients are biased

Impact of Measurement Error

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  - Even when the errors are completely random
- And we have only considered relatively simple scenarios
- In the words of Nugent et al. (2000: 60):
  - *"Measurement error is, to borrow a metaphor, a gremlin hiding in the details of our research that can contaminate the entire set of estimated regression parameters"*

Impact of Errors in Police Data

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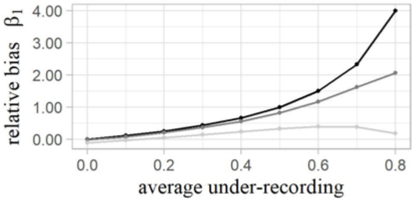

Random error: none, moderate (sd=0.04), substantial (sd=0.08)

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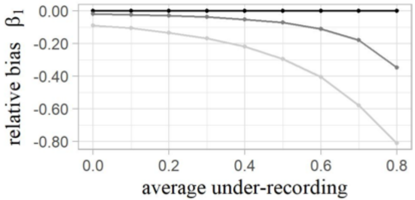

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- We should always aim to improve data collection processes
- When that is not possible/sufficient we should adjust for the impact of measurement error
- We have seen how in some simple settings we can anticipate - and therefore adjust - that impact
- When we can't trace out the impact of measurement error algebraically we need to use other methods

Adjustment Methods

- Most adjustment methods require additional forms of data
  - Multiple reflective indicators (latent variable models)
  - Instrumental variables (two stage processes)
  - A validation subsample (multiple imputation)
  - Repeated observations (regression calibration)

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- Others can be used when all you have is an educated guess (sensitivity analysis)
  - Bayesian adjustments (Gustaffson, 2003)
  - Multiple overimputation (Blackwell et al., 2017)
  - SIMEX (Cook & Stefanski, 1994)
  - Simulations (the *RCME* package Pina-Sánchez et al., 2022)

Sensitivity Analysis Using RCME

- We want to estimate the effect of collective efficacy on criminal damage rates, controlling for a few area characteristics
  - $log(Y^*) = \alpha + \beta_1 X + \beta_k Z_k + \epsilon$
  - Population: 982 postcode areas in London
- We estimate it first using police data (naive model), and we re-estimate it using adjusted crime rates (sensitivity analysis)

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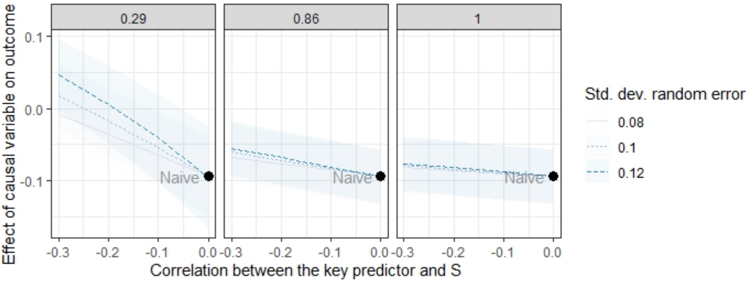
- We estimate it first using police data (naive model), and we re-estimate it using adjusted crime rates (sensitivity analysis)
- We simulate different combinations of measurement error to obtain multiple sets of 'adjusted' crime rates
  - We consider values for S (systematic error), R (random error), and D (differential errors)
  - which we derive from the Crime Survey for England and Wales
  - The measurement error term: $U = S + R + D \cdot X$
  - The adjusted crime rates: Y^*/U

The Naive Model

	outcome variable: log(criminal damage)	
	Using police data	Adjusting for measurement error
constant	0.88***	
collective efficacy	**-0.09*****	**?**
% white British	0.11***	
% unemployed	0.14***	
median age	−0.12***	
R^2	0.23	
N	982	

- We re-estimate that model considering different errors
  - $S = (1, 0.86, 0.29)$
  - $R = (0.08, 0.1, 0.12)$
  - $D = (0, -0.1, -0.2, -0.3)$

The *RCME* Adjustment



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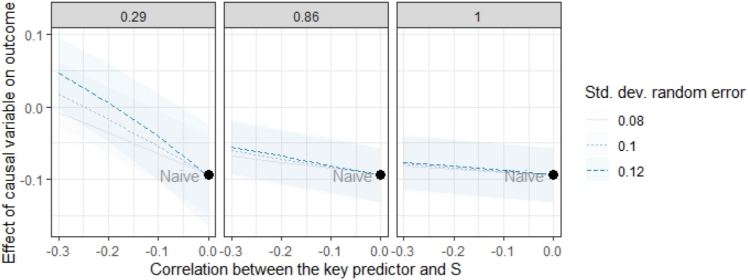

- The effect size attributed to collective efficacy appears robust, unless...
  - in the presence of low recording rates, variable recording rates across areas, and some minor differential error
  - That is however the most likely scenario according to victimisation surveys

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  - And yet, we do very little about it
- Besides improving data collection processes, we also need to employ adjustments methods
  - Many require additional data, others are quite complex
  - But there are a few methods (e.g. simulations) that are simple enough, and can be used even as a sensitivity analysis tool

Thank you!